**Chapter 2: Introduction to proof Test A** Name: Eden Tomes \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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|  | Identify the following numbers as either rational or irrational.  (a) −3 Rational  (b) 67.676 776 777 677 7… Irrational  (c) Rational  (d) Irrational |  |  |
|  | Express the following common fractions as decimal fractions.  (a)  (b) ???? |  |  |
|  | (a) Express  as a common fraction.  (b) Express  as a common fraction |  |  |
|  | Determine if the following are true statements  (a) If *x* is a positive integer, then  is a positive integer. True  (b) If , then . True  (c) If today is Thursday, then yesterday was Friday. False  (d) If  is rational, then *x* and *y* are integers. True |  |  |
|  | For each of the following, write the converse and determine if implication () or equivalence () is the more appropriate symbol.  (a) If *x* is an even integer, then  is an even integer. TRUE If 3x is an even integer, then x is an even integer : FALSE Implication is appropriate  (b) If , then    Equivalence is appropriate  (c) If *x* is odd, then 2*x* is even. TRUE If 2x is even, then x is odd. FALSE Implication is appropriate  (d) If , then . TRUE If , then . FALSE Implication is appropriate |  |  |
|  | Identify the negation of each of the following statements.  (a)  (b) The number is prime.  The number is composite.  (c) 1 = 5 1 ≠ 5  (d) The number is even. The number is odd. |  |  |
|  | Identify the negation of each of the following statements.  (a) The numbers are even or prime. The numbers are odd AND composite.  (b) The numbers are multiples of 5 and 3. The numbers are not multiples of 5 OR 3  (c) If *x* is even, then *x* + 1 is odd. If x is even, then x + 1 is even.  (d) If , then . If , then |  |  |
|  | Identify the contrapositive of the following statements  (a) If *x* is even, then *x* + 1 is odd. If x + 1 is even, then x is odd.  (b) If *x* is odd, then  is odd. If x2 is even, then x is even.  (c) If it is raining, then today is Monday. If today is not Monday, then it is not raining.  (d) If two angles are congruent, then they have the same measure. If two angles have different measures, then they are incongruent.  (e) If a shape is a rectangle, then it has two pairs of congruent sides. If a shape does not have two pairs of congruent sides, then it is not a rectangle.  (f) If a shape is an equilateral triangle, it has three congruent sides. If a shape does not have three congruent sides, then it is not an equilateral triangle. |  |  |
|  | Identify counter examples to disprove the following  (a) If *x* is a negative integer, then  is a negative integer. Counterexample: x is a negative integer and x2 is a positive integer. Let x = -1, meaning x2 = (-1)2 = 1 Therefore, x is a negative integer and x2 is not a negative integer, disproving the statement.  (b) If *x* and *y* and integers, then  is an integer. Counterexample: x and y are integers, and x/y is not an integer. Let x = 1 and y = 2, meaning x/y=1/2=0.5 Therefore, x and y are integers, but x/y is not an integer, disproving the statement.  (c) If  is a positive integer, then *x* is a positive integer. Counterexample: x2 is a positive integer and x is a negative integer. Let x2 = 4 and x = -2, meaning x2= (-2)2 = 4 Therefore, x2 is a positive integer, but x is a negative integer, disproving the statement.  (d) If , then . Counterexample: x2 > 1 and x ≤ 2 Let x2 = 2, meaning x = √2 ≈ 1.4 Therefore x2 is greater than 1, but x is less than / equal to 2, disproving the statement. |  |  |

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|  | Consider the statement ‘If  is even, then *x* is even.’  (a) Identify the contrapositive statement. If x is odd, then x2 is odd.  (b) Determine if the contrapositive statement is true. Let x = 3, meaning x2 = 32 = 9, therefore x is odd and x2 is odd, proving the contrapositive statement true.  (c) Determine if the original statement is true. The statement ‘If x2 is even, then x is even.’ is true because if the contrapositive of a statement is true, then so is the original statement.  (d) Determine if the converse statement is true. |  |  |

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|  | Prove that  is irrational. |  |  |

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|  | If a three-digit number is written down twice, to form a six-digit number, prove that the resulting number will have the numbers 7, 11 and 13 as factors. |  |  |
|  | Prove that there are infinitely many rational numbers. |  |  |
|  | Prove that it is impossible to find four different numbers *a*, *b*, *c* and *d* so that . |  |  |